3.7 Factoring a Sum or Difference of Cubes (Powers)

A Difference of Two Cubes

The following formula is called the *difference of cubes* identity.

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Ex 1. Use the difference of cubes identity to factor the following polynomial functions:

a) $x^3 - 8$

b) $27x^3 - 64$

c) $\frac{x^3}{27} - 125$

B Sum of Two Cubes

The following formula is called the *sum of cubes* identity.

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Ex 2. Use the sum of cubes identity to factor the following polynomial functions:

a) $x^3 + 1$

b) $8x^3 + 27$

c) $\frac{x^3}{64} + \frac{8}{27}$

C Difference of Two Powers

For any natural number n, the following identity is true:

$$a^n - b^n =$$
$$= (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + a^2b^{n-3} + ab^{n-2} + b^{n-1})$$

Ex 3. Factor as much as you can.

a) $x^2 - 4$

b) $x^4 - 16$

Ex 4. Use synthetic division to factor $x^5 - 32$.

D Sum of Two Powers

If n is an *odd* natural number, the following identity is true:

$$a^n + b^n =$$
$$= (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots \pm a^2 b^{n-3} \mp ab^{n-2} \pm b^{n-1})$$

Ex 5. Use synthetic division to factor completely.

a) $x^5 + 1$

b) $x^7 + 128$

Ex 6. Use different techniques to factor.

a) $x^6 - 1$

b) $x^{10} - 1$

c) $x^9 + 1$

Ex 7. Given that $a - b = 4$ and $ab = 2$, find $a^3 - b^3$.

Reading: Nelson Textbook, Pages 178-181
Homework: Nelson Textbook, Page 182: #2ac, 3ac, 5a, 6, 8